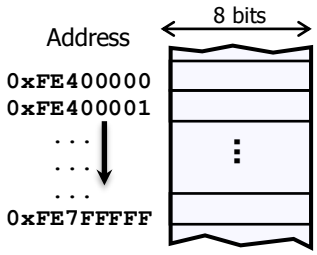
Homework 2

(Due date: October 4th @ 5:30 pm)
Presentation and clarity are very important! Show your procedure!

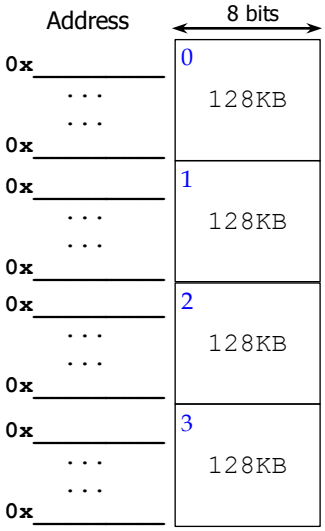
PROBLEM 1 (28 PTS)

a) What is the minimum number of bits required to represent: (2 pts)
   - ✓ 16385 symbols?
   - ✓ Memory addresses from 0 to 131072?

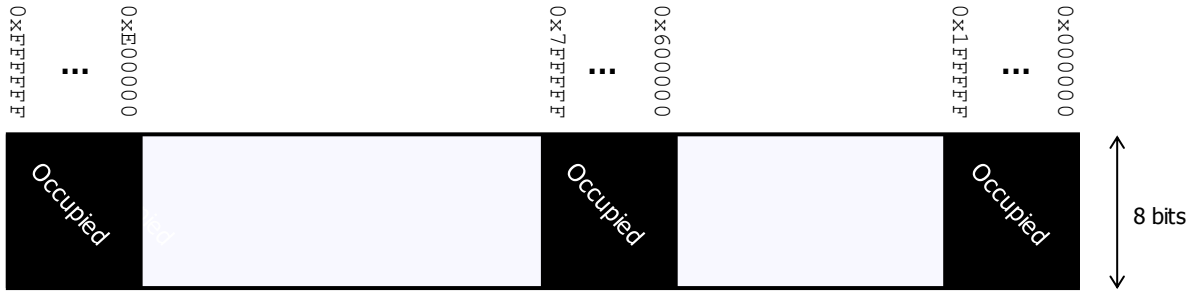
b) A microprocessor has a 32-bit address line. The size of the memory contents of each address is 8 bits. The memory space is defined as the collection of memory positions the processor can address. (6 pts)
   - What is the address range (lowest to highest, in hexadecimal) of the memory space for this microprocessor? What is the size (in bytes, KB, or MB) of the memory space? 1KB = 2^{10} bytes, 1MB = 2^{20} bytes, 1GB = 2^{30} bytes
   - A memory device is connected to the microprocessor. Based on the size of the memory, the microprocessor has assigned the addresses `0xFE400000` to `0xFE7FFFFF` to this memory device.
     - What is the size (in bytes, KB, or MB) of this memory device?
     - What is the minimum number of bits required to represent the addresses only for this memory device?



c) A microprocessor has a memory space of 512 KB. Each memory address occupies one byte. (8 pts)
   - What is the address bus size (number of bits of the address) of this microprocessor?
   - What is the range (lowest to highest, in hexadecimal) of the memory space for this microprocessor?
   - The figure to the right shows four memory chips that are placed in the given positions:
     - Complete the address ranges (lowest to highest, in hexadecimal) for each of the memory chips.



d) The figure below depicts the entire memory space of a microprocessor. Each memory address occupies one byte. (12 pts)
   - What is the size (in bytes, KB, or MB) of the memory space? What is the address bus size of the microprocessor?
   - If we have a memory chip of 2MB, how many bits do we require to address 2MB of memory?
   - We want to connect the 2MB memory chip to the microprocessor. Recall that a memory chip must be placed in an address range where every single address share some MSBs (e.g.: `0x600000` to `0x7FFFFF`). Provide a list of all the possible address ranges that the 2MB memory chip can occupy. You can only use any of the non-occupied portions of the memory space as shown below.



PROBLEM 2 (28 PTS)

- In ALL these problems (a, b, c), you MUST show your conversion procedure. **No procedure ≡ zero points**.

a) Convert the following decimal numbers to their 2's complement representations: binary and hexadecimal. (9 pts)
   - ✓ -136.6875, 207.65625, -128.5078125

b) Complete the following table. The decimal numbers are unsigned: (7 pts.)

Decimal	BCD	Binary	Reflective Gray Code
278			
		10101011	
			110101010
507			
	100110010101		
			10110101
		01010100	

c) Complete the following table. Use the fewest number of bits in each case: (12 pts.)

	REPRESENTATION		
Decimal	**Sign-and-magnitude**	**1's complement**	**2's complement**
-129			
		01010100	
			10101000
		11111	
			1000000
	1100111		

PROBLEM 3 (38 PTS)

a) Perform the following additions and subtractions of the following unsigned integers. Use the fewest number of bits n to represent both operators. Indicate every carry (or borrow) from c_0 to c_n (or b_0 to b_n). For the addition, determine whether there is an overflow. For the subtraction, determine whether we need to keep borrowing from a higher bit. (8 pts)

Example (n=8):

✓ 54 + 210

	c_8=1	c_7=1	c_6=1	c_5=1	c_4=0	c_3=1	c_2=1	c_1=0	c_0=0
54 = 0x36 =		0	0	1	1	0	1	1	0 +
210 = 0xD2 =		1	1	0	1	0	0	1	0
Overflow! →	1	0	0	0	0	1	0	0	0

✓ 77 - 194

	Borrow out! →	b_8=1	b_7=0	b_6=0	b_5=0	b_4=0	b_3=0	b_2=1	b_1=0	b_0=0
77 = 0x4D =			0	1	0	0	1	1	0	1 -
194 = 0xC2 =			1	1	0	0	0	0	1	0
			0	0	0	0	1	0	1	1

- ✓ 209 + 68
- ✓ 34 + 437
- ✓ 129 - 37
- ✓ 42 - 98

b) We need to perform the following operations, where numbers are represented in 2's complement: (24 pts)

- ✓ -98 + 256
- ✓ 206 + 309
- ✓ -257 + 256
- ✓ 105 - 62
- ✓ -127 - 36
- ✓ 246 + 31

- For each case:
  - ✓ Determine the minimum number of bits required to represent both summands. You might need to sign-extend one of the summands, since for proper summation, both summands must have the same number of bits.
  - ✓ Perform the binary addition in 2's complement arithmetic. The result must have the same number of bits as the summands.
  - ✓ Determine whether there is overflow by:
    i. Using c_n, c_{n-1} (carries).
    ii. Performing the operation in the decimal system and checking whether the result is within the allowed range for n bits, where n is the minimum number of bits for the summands.
  - ✓ If we want to avoid overflow, what is the minimum number of bits required to represent both the summands and the result?

c) Get the multiplication results of the following numbers that are represented in 2's complement arithmetic with 4 bits. (6 pts)
   - ✓ 1001x1011, 1010x0101, 1110x0110.

PROBLEM 4 (6 PTS)

- Complete the timing diagram (signals DO and $DATA$) of the following circuit. The circuit in the blue box is a 4-bit Binary to Gray Decoder. For example, if `T=1100`, then `DO=1010`.